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Author's address Hans Gersbach
 Alfred-Weber-Institut
 Grabengasse 14
 69117 Heidelberg
 Phone: +49-6221-543173
 E-mail: gersbach@wp.awi.uni-heidelberg.de

Armin Schmutzler
Sozialökonomisches Seminar
Universität Zürich
Blümlisalpstrasse 10
CH-8006 Zürich
Phone: +41-1-634 37 21
E-mail: arminsch@sozoec.unizh.ch

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Hans Gersbach and Armin Schmutzler

Alfred-Weber-Institut

Grabengasse 14

69117 Heidelberg

Germany

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Abstract:

We present a new approach to endogenizing technological spillovers. Firms choose continuous levels of a cost-reducing innovation before they engage in competition for each other's R&D-employees. Successful bids for the competitor's employee then result in higher levels of cost reduction. Finally, firms enter product market competition. We apply the approach to the long-standing debate on the effects of the mode of competition on innovation incentives. We show that incentives to acquire spillovers are stronger and incentives to prevent spillovers are weaker under quantity competition than under price competition. As a result, for a wide range of parameters, price competition gives stronger innovation incentives than quantity competition.

Keywords: Innovation Incentives, Spillovers, Product Market Competition

Endogenous Spillovers and Incentives to Innovate

1. Introduction

In both empirical and theoretical studies of the innovation process, the importance of knowledge spillovers has often been emphasized. Some authors have focused on the potential adverse consequences for innovation incentives (d'Aspremont and Jacquemin 1988, Kamien et al. 1992, Suzumura 1992, Henriques 1990, De Bondt et. al. 1992, Leahy and Neary 1997), others have examined adverse and beneficial effects of spillovers on economic growth (Romer 1986 and 1990, Aghion and Howitt 1992, Grossman and Helpman 1991). Recently, the effects of spillovers on agglomeration patterns have been analyzed (Baldwin et al. 1998).

This entire literature has used the convenient simplification that spillover levels are exogenous, that is, if one firm achieves a cost reduction, other firms receive a fixed proportion of this cost reduction in the spillover process, which they cannot influence through their decisions. Only recently have some authors tried to come to grips with the fact that the spillover level is endogenous: firms can engage in activities designed at preventing spillovers to competitors, and activities designed at obtaining spillovers from competitors (see Katsoulacas and Ulph 1996, Gersbach and Schmutzler 1997, Fosfuri et al. 1998, Ronde 1998).

We start from Gersbach and Schmutzler 1997 who have suggested that technological spillovers depend on the ability of firms to attract R&D workers of other firms and to prevent their own R&D workers from leaving the firm. We examine a game in which firms choose continuous levels of a cost-reducing innovation before they engage in competition for each other's R&D-employees.¹ Whether spillovers arise depends on the relative attractiveness of the contracts offered by the original employer and his competitors. Successful bids for the competitor's employee result in higher levels of cost reduction. Finally, firms enter product market competition, given the cost structure determined by innovation and spillover levels. Our main contributions are as follows:

First, we characterize necessary and sufficient conditions for equilibria in which one-way and two-ways technological spillovers occur. We show how the extent of spillovers depends on the mode of competition.

¹ Gersbach and Schmutzler (1997) only consider 0-1 decisions ("innovate" versus "do not innovate") which is not suitable to study innovation incentives.

Second, we show that taking the endogeneity of spillovers into account affects familiar results on the incentives for innovation in a systematic way. We illustrate this point by taking up the long-standing debate on the effects of the mode of competition on innovation incentives. In both IO and growth models, various authors have investigated how tough competition (à la Bertrand) and soft competition (à la Cournot) differ in this respect. Most of these authors (Brander and Spencer 1983, Delbono and Denicolò 1990, Bester and Petrakis 1993) have argued in a world without spillovers. Qiu (1997) has made the important point that, with exogenous spillovers, incentives for innovation become relatively weaker in the Bertrand case than in the Cournot case.² In this paper, we show that, with endogenous spillovers, this argument becomes weaker and, in many cases, is reversed, the reason being that incentives to acquire spillovers are stronger and incentives to prevent spillovers are weaker under quantity competition. Under Bertrand competition there are no negative effects of spillovers on incentives to produce knowledge. As a result, an innovating firm has to worry less about spillovers under price competition than under quantity competition, and, consequently, for wide ranges of parameters, price competition gives stronger innovation incentives than quantity competition. Our results suggest that weakening product market competition in order to spur innovations cannot be justified when spillovers are endogenous.

The paper is organized as follows. In Section 2, we present a three-stage duopoly model. Firms first choose innovation levels, then compete for knowledge by making wage offers to each others R&D employees. Finally, they compete on the product market. In Section 3, we give general conditions for the extent of spillovers, assuming given innovation levels. Section 4 applies these results to the discussion of relative innovation incentives in the Cournot and Bertrand cases. Section 5 concludes.

² Another related paper with spillovers is Aghion et al. (1997).

2. The Model

We consider a three-stage game. There are two firms, $i = 1, 2$. Initially, firms have constant marginal costs c . In period 1, they can carry out an innovation that reduces marginal costs by x_i . Following Qiu (1997), innovation costs are $k(x_i) = \nu x_i^2$, where $\nu > 0$.

To carry out the innovation, each firm has to hire an R&D worker. In period 2, firms bid for each other's R&D workers: firm i offers wages w_{ij} , $j \neq i$ for firm j 's R&D worker and w_{ii} for the own R&D worker. An R&D worker from firm j is assumed to switch firms if $w_{ij} > w_{jj}$, otherwise he continues to work for his own firm. To attract the worker from firm j , firm i needs to offer a wage which is higher than w_{jj} by the smallest possible currency unit.

We assume that wage contracts offered to R&D employees can be conditioned on the knowledge of both R&D-employees and thus on the relative performance of the R&D employees. The contractability assumption involves two elements. On the one hand, the wage offer depends on the knowledge of the R&D-person himself, on the other hand it depends on the knowledge of the other firm's employee. While the first element is not critical, the second element requires that firms can observe and verify the knowledge at beginning of their wage bids or, more plausibly, when employees have accepted wage contracts and enter firms. In Gersbach and Schmutzler (1997), we discuss in detail how this assumption can be justified.

If an R&D worker moves to firm i , this firm obtains a further cost reduction x_j thanks to knowledge spillovers, so its marginal production costs are $c_i = c - x_i - x_j$. Hence, we assume that the knowledge necessary to reduce costs is completely transferable to other firms; that is, if spillovers arise, they are perfect: if a firm can motivate the R&D person of the other firms to move, this employee will be able to replicate the original cost reduction in his new firm.

If firm i does not obtain the services of worker j , production costs remain at $c_i = c - x_i$. In period 3, product market competition takes place. We shall suppose the two firms produce homogeneous goods. The inverse market demand is given by

$$(1) \quad p = a - q_i - q_j; \quad i, j = 1, 2; \quad i \neq j; a > 0$$

We assume that marginal innovation costs are sufficiently high.

$$\mathbf{A1:} \quad \nu > \max \left\{ \frac{2a - c}{9c}, \frac{a - c}{2c} \right\}.$$

This assumption will later be seen to imply that $c > x_i + x_j$, which is necessary and sufficient for positive marginal costs. We consider both price and quantity competition. In either case, a unique equilibrium of the period 3 subgame exists for each cost vector (c_i, c_j) determined in period 1 and 2. We denote the resulting profits in the product market for firm i as $\pi(c - c_i, c - c_j)$. We denote by $\pi^n(c - c_i, c - c_j)$ the net profit of firm i , defined as $\pi(c - c_i, c - c_j)$ minus wages paid to the R&D persons who will be employed in period 2.

The assumptions of perfect spillovers and homogenous goods simplify the analysis. They are also chosen because Qiu (1997) has shown that Cournot competition is more likely to give stronger innovation incentives when goods are close substitutes and spillovers are high. Hence, with our assumptions we can make the point that, even in a setting that satisfies these characteristics in the best possible way, the opposite result arises for endogenous spillovers.

3. Bilateral Spillovers

We now consider the spillover game, that is, the subgame starting with given first-period investment levels x_i, x_j . We say that the resulting spillovers are bilateral if each firm acquires the services of the other firm's worker, unilateral if only one firm acquires the services of the other firm's worker. We characterize the conditions under which bilateral spillovers occur. Here and in the following, we assume that the firms do not play weakly dominated strategies.

To simplify the exposition, we shall always neglect the smallest currency unit. Hence, if two wages are identical in equilibrium, it is understood that the firm that wins the wage bid offers the equilibrium wage plus the smallest currency unit.

Proposition 1:

(a) *An equilibrium with bilateral spillovers exists if and only if the following conditions hold.*

$$(i) \quad 2\pi(x_i + x_j, x_i + x_j) \geq \pi(x_i + x_j, x_i) + \pi(x_i, x_i + x_j); \quad i, j = 1, 2; \quad i \neq j$$

$$(ii) \quad \pi(x_i + x_j, x_i + x_j) + \pi(x_i + x_j, x_j) \geq \pi(x_i, x_j) + \pi(x_i + x_j, x_i); \quad i, j = 1, 2; \quad i \neq j$$

(b) *In this equilibrium, wages are $w_{ji} = w_{ii} = \pi(x_i + x_j, x_j) - \pi(x_i + x_j, x_i + x_j)$ and net profits*

$$\text{are } \pi^n = 2\pi(x_i + x_j, x_i + x_j) - \pi(x_i + x_j, x_i) \text{ for } i, j = 1, 2; \quad j \neq i$$

The proof is given in the appendix. There, we also prove our second result.

Proposition 2:

(a) *An equilibrium with bilateral spillovers and an equilibrium with unilateral spillovers never exist simultaneously, except in the non-generic case that*

$$2\pi(x_i + x_j, x_i + x_j) = \pi(x_j, x_i + x_j) + \pi(x_i + x_j, x_j)$$

for at least one $i \in \{1, 2\}$ and $j \neq i$.

(b) *An equilibrium without spillovers exists if and only if the following conditions hold for $i, j = 1, 2; i \neq j$:*

$$(A) \pi(x_i + x_j, x_j) - \pi(x_i + x_j, x_i) \geq \pi(x_i + x_j, x_i + x_j) - \pi(x_j, x_i)$$

$$(B) \pi(x_i, x_j) + \pi(x_j, x_i) \geq \pi(x_i, x_i + x_j) + \pi(x_i + x_j, x_i)$$

Together, proposition 1 and 2 allow us to isolate the conditions under which only bilateral spillovers occur.

In the next section we will apply propositions 1 and 2 to price and quantity competition. We will show that, for Cournot competition, only bilateral spillovers occur if innovations are not too large. Under Bertrand competition, only unilateral spillovers can occur. We shall use this result to derive the subgame perfect equilibrium of the innovation game which provides a comparison of innovation incentives under Bertrand and Cournot competition when technological spillovers are endogenized.

4. The Nature of Competition and Innovation Incentives

There exists a long-standing debate on the effects of the mode of competition on innovation incentives. In both IO and growth models, various authors have investigated how tough competition (à la Bertrand) and soft competition (à la Cournot) differ in this respect. Most authors (Brander and Spencer 1983, Delbono and Denicolò 1990, Bester and Petrakis 1993, Aghion et al. 1997) have argued in a world without spillovers and some (e.g. Qiu 1997) have considered exogenous spillovers [see also the discussions in Bonnano and Haworth 1998 and Gerowski 1995]. We complement this discussion by endogenizing technological spillovers. We will show that, with endogenous spillovers, Bertrand competition yields higher innovation incentives than Cournot competition for sufficiently high innovation costs, which differs from the case of exogenous spillovers.

4.1. Bilateral Spillovers in the Cournot Case

We first apply propositions 1 and 2 to the Cournot case. Straightforward applications of the

standard result that $\pi(c_i, c_j) = \frac{(a - 2c_i + c_j)^2}{9}$ can be used to characterize the conditions for which the second stage yields bilateral spillovers.

Corollary 1: *In the Cournot case, the following statements hold:*

- (a) *An equilibrium with bilateral spillovers exists if and only if $2(a - c) \geq 3x_i - 2x_j$ ($i, j = 1, 2; i \neq j$); there is only one such equilibrium.*
- (b) *If $2(a - c) > 3x_i - 2x_j$, spillovers are bilateral for every pure strategy equilibrium.*
- (c) *In a bilateral spillover equilibrium, net profits are*

$$(2) \quad \pi^u = 2 \frac{(a - c + x_i + x_j)^2}{9} - \frac{(a - c + 2x_j + x_i)^2}{9}.$$

Proof:

(a) $2(a - c) \geq 3x_i - 2x_j$ is condition (i) of proposition 1. It also implies condition (ii) of proposition 1, because in the Cournot case with linear demand, this condition becomes $2(a - c) \geq 3x_i - 6x_j$ for $i, j = 1, 2; j \neq i$. By proposition 1(b), there is only one bilateral spillover equilibrium. Hence, part (a) follows. $2(a - c) \geq 3x_i - 6x_j$ also implies that an equilibrium without spillovers does not exist, by checking proposition 2 (b) (A). By proposition 2 (a) there is no equilibrium with unilateral spillovers, completing part (b) of the corollary. Part (c) follows directly from part (b) of proposition 1. (q.e.d.)

We now use our results to delineate parameter regions for which there is unique subgame perfect equilibrium of the full game such that there are bilateral spillovers.

Proposition 3: *Suppose $v \geq 49/36 \approx 1.36$*

Then, a unique subgame perfect equilibrium exists in the Cournot case.³ In this equilibrium,

$$(3) \quad x_1^c = x_2^c = \frac{a - c}{9v - 1}.$$

³ The condition $v > 49/36$ is sufficient, but not necessary. Necessary conditions could be calculated by modifying the proof and examining the conditions under which unilateral spillover equilibria and no spillover equilibria arise. This would involve tedious derivations, and it would not provide any further insights into the issues in this paper.

Proof: see appendix.

4.2. Spillovers under Bertrand Competition

We now consider the case of price competition. Here we need to examine unilateral spillovers explicitly. In this case, firm i will obtain

$$(4) \quad \pi(x_i, x_j) = \max\{(x_i - x_j)(a - c + x_j), 0\}$$

provided that $c - x_j$ is not greater than firm i 's monopoly price. For linear demand, this condition amounts to

$$(5) \quad x_i \leq a - c + 2x_j.$$

Proposition 4: *Suppose firms are labeled so that $x_1 \geq x_2$. Also suppose inequality (5) holds. In the Bertrand case, there exists an equilibrium involving unilateral spillovers. For this equilibrium, the following conditions hold:*

$$(6) \quad \pi(x_1 + x_2, x_2) - \pi(x_1, x_1 + x_2) \geq w_{11} + w_{12} = \pi(x_1 + x_2, x_1) - \pi(x_2, x_1 + x_2) \text{ and}$$

$$(7) \quad w_{12} \leq \pi(x_1 + x_2, x_2) - \pi(x_1, x_2)$$

Net profits are

$$(8) \quad \pi(x_1 + x_2, x_2) - \pi(x_1 + x_2, x_1) \geq 0 \text{ for firm 1, } 0 \text{ for firm 2.}^4$$

If $x_1 > x_2$, the spillovers flow from firm 2 to firm 1; if $x_1 = x_2$, the direction is indeterminate.

Finally, any pure strategy equilibrium involves unilateral spillovers and satisfies (6)-(8).⁵

Proof: see appendix. Using this result, we can immediately analyze the equilibrium structure of the full game, provided marginal innovation costs are not too low.

Proposition 5: *Suppose $v \geq 1/4 + \sqrt{3/8}$.*

Then, except for relabeling, there exists exactly one equilibrium in the Bertrand case. In this equilibrium,

⁴ In particular, if $x_1 = x_2$, both firms earn zero profits.

⁵ In Gersbach and Schmutzler (1997), we show in a similar setting that this result does not depend on the homogeneity of the firm's outputs.

$$x_1^B = \frac{a-c}{2v}; x_2^B = 0.$$

Proof: see appendix.

4.3. Comparison of Innovation Incentives

Putting together Propositions 3 and 5, it immediately follows that innovation incentives are stronger for price competition than for quantity competition, provided innovation costs are sufficiently high.

Proposition 6: Suppose $v \geq 49/36 \approx 1.36$.

Then, for suitable choice of firm indexes,

$$x_1^B = \frac{a-c}{2v} > 2\frac{a-c}{9v-1} = 2x_1^c = 2x_2^c > x_2^B = 0.$$

In particular, total investment is higher for Bertrand competition than for Cournot competition.⁶

This result is the main conclusion of our analysis. For Bertrand competition, having the same costs as the competitor is not preferable to having higher costs, whereas having lower costs is preferable to having identical costs. Therefore, incentives to acquire spillovers are low relative to the incentives to prevent them, and the fear of spillovers does not reduce innovation incentives. As this logic does not apply to Cournot competition, ignoring the endogeneity of spillovers overstates the innovation incentives in the Cournot case relative to the Bertrand case. With endogenous spillovers, therefore, innovation incentives may be stronger for Bertrand competition, even when, for exogenous spillovers, the converse would be true.

5. Conclusion

In this paper, we introduced a particular framework for the analysis of innovation incentives in the presence of endogenous spillovers. As an application, we showed that, compared to the case of exogenous spillovers, innovation incentives are strengthened in the Bertrand case relative to the Cournot case.

The results have been derived under the simplifying assumption of homogeneous goods and perfect spillovers. With suitable modifications, the above arguments still hold with these

⁶ Note from the proof of proposition 5 that the condition $v \geq 49/36$ is sufficient but not necessary.

assumptions relaxed. For instance, as long as product differentiation is not too strong, incentives to obtain spillovers are still fairly low, and by similar reasoning as above, innovation incentives are hardly affected by the prospect of spillovers when competition is in prices; accordingly, familiar results on the relation between innovation incentives in the Bertrand and Cournot case may be reversed.

Our model could be extended in various directions. The analysis would be much more complicated in the case that there are $n > 1$ R&D employees in each firm. This would introduce competing effects. On the one hand, it may be hard for firms to avoid spillovers if the knowledge of one worker can substitute for the knowledge of others - competitors may only have to attract a small number of employees out of a large group to obtain spillovers. On the other hand, no worker can easily appropriate the information rent because of the competition from other workers.⁷ The net effect compared with the present situation is unclear. Compared with the exogenous case, however, it still remains true that appropriability is easier to satisfy with endogenous spillovers. Also, because the incentives for firms to acquire spillovers are particularly small in the Bertrand case, we conjecture that it is still true that endogeneity of spillovers strengthens incentives for innovation in this case relative to the Cournot case. The proof of this general conjecture will be left for future research.

⁷ This competition is weakened, however, by the possible collusion between workers.

Appendix:

Proof of Proposition 1:

First, note that in equilibrium $w_{ij} = w_{ji}$ for $i, j = 1, 2; i \neq j$: $w_{ij} \geq w_{ji}$ is required for bilateral spillovers, and $w_{ij} \leq w_{ji}$ is required because otherwise firm i could decrease its wage offer and still obtain spillovers.

Also, in a bilateral spillover equilibrium $w_{ii} = w_{ji} = \pi(x_i + x_j, x_j) - \pi(x_i + x_j, x_i + x_j)$; for lower values of w_{ji} firm i would be prepared to increase its wage offer w_{ii} to avoid spillovers; for higher values of w_{ii} firm i would be playing a weakly dominated strategy. It remains to be shown that these wages constitute an equilibrium.

- (a) It is not a profitable deviation for firm i to set w_{ij} so low that both employees work for the competitor: by condition (i), the resulting loss of profits, $\pi(x_i + x_j, x_i + x_j) - \pi(x_i, x_i + x_j)$ is at least as high as the reduction in the wage bill, $w_{ij} = \pi(x_i + x_j, x_j) - \pi(x_i + x_j, x_i + x_j)$.
- (b) It is not a profitable deviation for firm i to set w_{ij} lower and w_{ii} slightly higher, so that there are no spillovers: From condition (ii), we immediately obtain $\pi(x_i + x_j, x_i + x_j) - w_{ij} \geq \pi(x_i, x_j) - w_{ii}$.
- (c) It is not a profitable deviation for firm i to set w_{ii} so high that it avoids spillovers to the competitor (while obtaining spillovers itself):

$$\pi(x_i + x_j, x_i + x_j) \geq \pi(x_i + x_j, x_j) - w_{ii} \text{ holds with equality.}$$

Proof of Proposition 2:

- (a) For unilateral spillovers, one firm, say firm 2, must weakly prefer not attracting the competitor's worker, while firm 1 prefers to keep its own worker. This results in

$$\pi(x_1 + x_2, x_1 + x_2) - \pi(x_2, x_1 + x_2) \leq w_{21} = w_{11} \leq \pi(x_1 + x_2, x_2) - \pi(x_1 + x_2, x_1 + x_2),$$

which contradicts condition (i) of proposition 1, except if

$$2\pi(x_1 + x_2, x_1 + x_2) = \pi(x_2, x_1 + x_2) + \pi(x_1 + x_2, x_2).$$

- (b) By similar reasoning as in proposition 1, an equilibrium without spillovers requires

- (i) $w_{ji} = w_{ii} = \pi(x_i + x_j, x_i) - \pi(x_j, x_i)$
- (ii) $\pi(x_i, x_j) - w_{ii} \geq \pi(x_i + x_j, x_i + x_j) - w_{jj}$
- (iii) $\pi(x_i, x_j) - w_{ii} \geq \pi(x_i, x_i + x_j)$
- (iv) $\pi(x_i, x_j) \geq \pi(x_i + x_j, x_j) - w_{jj}$.

Inserting (i) in (ii) - (iv) and rearranging gives the result, using conditions (A) and (B).

Proof of Proposition 3:

First note that, by corollary 1(a) and 1(b), for the proposed values of x_1^c and x_2^c , and $v > 1/6$, there is a unique pure strategy equilibrium of the resulting subgame, and spillovers are bilateral in this equilibrium. By corollary 1(c), therefore, a Nash equilibrium with bilateral spillovers, requires

$$\max_{x_i} \left\{ 2 \frac{(a - c + x_i + x_j)^2}{9} - \frac{(a - c + 2x_j + x_i)^2}{9} - vx_i^2 \right\}, \text{ which implies } x_1^c = x_2^c = \frac{a - c}{9v - 1}.$$

Note that assumption A1 implies $v > (2a - c)/9c$ and hence $x_1^c + x_2^c < c$.

To test for subgame perfection, we need to check that it is not worthwhile for firm i to raise its investment level so much that firms are not in the bilateral spillover regime in period 2. By corollary 1b, leaving the bilateral spillover regime would require $x_i > 2(a - c + x_j^c)/3$, where

$$x_j^c = \frac{a - c}{9v - 1}.$$

In the following, we show that for sufficiently high marginal investment costs raising x_i above this critical level would yield negative total profits.⁸ In the most favorable case that firm 1 would obtain all the knowledge of firm 2 at zero costs and could prevent spillovers to firm 2 at zero cost, the deviation profit is given by:

$$\frac{(a - c + 2(x_i + x_j^c) - x_j^c)^2}{9} - vx_i^2$$

Straightforward but lengthy derivations show that this expression is negative if and only if

$$x_i > x_i^0 = \frac{9v(a - c)}{(9v - 4)(9v - 1)} \{2 + 3\sqrt{v}\}.$$

Hence, no matter which spillover equilibrium occurs for $x_i > x_i^0$, firm 1 will never choose an innovation level above x_i^0 .

⁸ We are implicitly assuming that $x_i + x_j^c \leq c$; for otherwise this deviation is not feasible anyway.

It suffices to show that for $x_i < x_i^0$, the firms remain in the bilateral spillover regime. This is true if $x_i^0 < 2(a - c + x_j^c)/3$. Inserting x_i^0 shows that this condition holds for

$$6 + 9\sqrt{v} \leq 2(9v - 4)$$

which implies after some manipulations

$$v \geq 49/36 \approx 1.36.$$

Hence, for $v \geq 1.36$, by corollary 1 no firm wants to invest so much to get out of the equilibrium with bilateral spillovers since profits would be negative in the best possible case. Therefore, the equilibrium is subgame perfect. (q.e.d.)

Proof of Proposition 4:

The proposed equilibrium exists by (i) - (vi) below.

(i) Because $w_{11} + w_{12} = \pi(x_1 + x_2, x_1) - \pi(x_2, x_1 + x_2) = \pi(x_1 + x_2, x_1)$ firm 2 does not want to attract both workers: this would leave net payoff unaffected.

(ii) For firm 2 attracting only the other firm's worker would lead to payoffs

$$\pi(x_1 + x_2, x_1 + x_2) - w_{21} \leq 0.$$

(iii) Avoiding spillovers would lead to net payoffs $\pi(x_2, x_1) - w_{22}$ for firm 2. If $x_2 \leq x_1$, this expression is also non-positive.

(iv) Firm 1 makes no deviation where both firms obtain spillovers and both firms end up with equal costs, as it would lose its profit.

(v) Similarly, firm 1 does not let firm 2 have both workers, because of

$$\pi(x_1 + x_2, x_2) = \pi(x_1 + x_2, x_2) - \pi(x_1, x_1 + x_2) \geq w_{11} + w_{12}.$$

(vi) Firm 1 does not refrain from hiring the other firm's worker because, by (7),

$$\pi(x_1, x_2) \leq \pi(x_1 + x_2, x_2) - w_{12}.$$

Also there is no equilibrium with unilateral spillovers that does not satisfy (6) and (7). By (i), (v) and (vi), the inequalities on wages given in (6) and (7) are necessary. In addition, the equality

$$w_{11} + w_{12} = w_{12} + w_{22} = \pi(x_1 + x_2, x_1) - \pi(x_2, x_1 + x_2) \text{ holds,}$$

for otherwise firm 2 would be offering more for the two workers than they are worth, that is, it would play a weakly dominated strategy. There is no equilibrium with bilateral spillovers, as this would lead to zero profits for both firms, and no firm would be willing to pay a positive

amount to obtain the competitor's worker. There is no equilibrium without spillovers: Such an equilibrium would require $w_{ii} = w_{ji}$. Supposing that $x_2 < x_1$, firm 2 would only pay $w_{22} = 0$. Increasing w_{12} slightly above zero would increase profits for firm 1.

Proof of Proposition 5:

- (i) First note that assumption A1 implies $(a - c) / 2v < c$, so that $x_1^B < c$.
- (ii) There is no equilibrium with $x_1 = x_2 > 0$: both firms would obtain zero profits in the product market; hence they would be better off not investing at all.
- (iii) There is no equilibrium with $x_1 = x_2 = 0$. Both firms obtain zero profits in the product market; using proposition 4, a small cost reduction x_1 would yield total payoffs $x_1(a - c) - vx_1^2$. For small x_1 , this is positive.
- (iv) There is no equilibrium with $x_1 > x_2 > 0$: firm 2 obtains zero profits in the product market, and hence, subtracting investment costs, negative total payoffs, so it would be better off setting $x_2 = 0$.
- (v) The proposed values of x_i are equilibrium choices. We first note that x_1^B maximizes $x_1(a - c) - vx_1^2$. Moreover, x_1^B is too low for firm 1 to obtain the monopoly profit, since condition (5) $x_1^B \leq a - c$ is satisfied for $v \geq \frac{1}{2}$. Therefore, $\pi(x_1, 0) = x_1(a - c)$ and x_1^B maximizes $\pi(x_1, 0) - vx_1^2$.

Next, we need to check that firm 1 has no incentive to select x_1 so high to obtain monopoly profits. Profits in the monopoly case when $x_1 > a - c$ would amount to

$$\frac{(a - c + x_1)^2}{4} - vx_1^2.$$

Maximizing with respect to x_1 yields $x_1 = \frac{a - c}{4v - 1}$ which is a contradiction to

$x_1 > a - c$ for $v \geq \frac{1}{2}$. Hence, firm 1 does not want to become a monopolist. Finally, we

have to show that firm 2 cannot set x_2 so high that it obtains a positive profit. We have to distinguish two cases. First, we assume that firm 2 does not become a monopolist by leapfrogging x_1^B . According to proposition 4 firm 2 will obtain spillovers from firm 1, and its total payoff is given by:

$\pi(x_1^B + x_2, x_1^B) - \pi(x_1^B + x_2, x_2) - vx_2^2 = x_2(a - c + x_1^B) - x_1^B(a - c + x_2) - vx_2^2$. The optimal value of x_2 obtainable in this way is given by $x_2 = \frac{a - c}{2v} = x_1^B$.

Since $\pi(x_1^B + x_2, x_1^B) - \pi(x_1^B + x_2, x_2) = 0$ for $x_2 = x_1^B$ firm 2 would have negative total payoff equal to $-vx_2^2$. Thus, firm 2 has no incentive to innovate itself.

Second, we show that firm 2 does not want to set x_2 so high that it will sell at the monopoly price. By (5), it would set the monopoly price if

$x_2 \geq a - c + 2x_1^B = (a - c)\left(\frac{v+1}{v}\right) = \bar{x}_2$. After choosing x_2 so high, it would obtain

$\pi(x_1^B + x_2, x_1^B) - \pi(x_1^B + x_2, x_2)$. $\pi(x_1^B + x_2, x_1^B)$ is the monopoly profit corresponding

to costs of $c - x_1^B - x_2$, i.e. $\frac{(a - c + x_1^B + x_2)^2}{4}$. $\pi(x_1^B + x_2, x_2)$ is equal to

$x_1^B \cdot (a - c + x_2)$. Hence, the optimal deviation profit involving monopoly pricing is

$\max_{x_2 \geq \bar{x}_2} \frac{(a - c + x_1^B + x_2)^2}{4} - \frac{4x_1^B(a - c + x_2)}{4} - \frac{4vx_2^2}{4}$. An interior solution requires

$2(a - c + x_1^B + x_2) - 4x_1^B - 8vx_2 = 0$. The second-order-condition is fulfilled for $v > \frac{1}{4}$.

We obtain $x_2 = \frac{x_1^B - (a - c)}{1 - 4v}$.

Finally, we show that if this expression is larger than \bar{x}_2 , we obtain a contradiction and hence firm 2 does not set the monopoly price. To this end, note that

$$\frac{x_1^B - (a - c)}{1 - 4v} = \frac{\frac{a - c}{2v} - (a - c)}{1 - 4v} = (a - c) \left(\frac{1 - 2v}{(1 - 4v)2v} \right). \text{ Hence, } x_2 = \frac{x_1^B - (a - c)}{1 - 4v} > \bar{x}_2$$

implies

$$(a - c) \frac{1 - 2v}{(1 - 4v)2v} > (a - c) \left(\frac{v+1}{v} \right) \text{ or } \frac{1 - 2v}{(1 - 4v)} > 2(v+1). \text{ For } v > \frac{1}{4}, \text{ this is equivalent to}$$

$$v^2 + \frac{1}{2}v - \frac{1}{8} < 0 \text{ and hence } v < \frac{1}{4} + \sqrt{\frac{3}{8}} \text{ which is a contradiction to our assumption.}$$

(q.e.d.)

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